9-1 day 4 Alternating Series

Learning Objectives:

I can use the alternating series test to determine whether an infinite alternating series converges or diverges

I can use the alternating series remainder to approximate the sum of an alternating series

I can classify an convergent alternating series as absolutely or conditionally convergent

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Alternating Series

$$\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n-1} = 1 + -\frac{1}{2} + \frac{1}{4} + -\frac{1}{8} + \frac{1}{16} + \ldots$$

or

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{2^{n-1}} = 1 + -\frac{1}{2} + \frac{1}{4} + -\frac{1}{8} + \frac{1}{16} + \ldots$$

Are alternating series because the signs of the terms alternate.

Alternating Series Test

Let $a_n>0$ for all n. The alternating series

$$\sum_{n=1}^{\infty}(-1)^n a_n \quad \text{or} \quad \sum_{n=1}^{\infty}(-1)^{n+1} a_n$$

will converge if the following two conditions are met: $\lim_{n\to\infty} a_n = 0$ $\quad |a_{n+1}| \le |a_n|$

1.) 2.)

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* Note – this means that the alternating harmonic series converges and the harmonic series doesn't.

Alternating Harmonic series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$$

$1 + \frac{-1}{2} + \frac{1}{3} + \frac{-1}{4} + \frac{1}{5} + \cdots$

$\lim_{n \to \infty} \frac{1}{n} = 0$ converges

converges

Harmonic Series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

diverge p-test

diverges

Ex1. Determine if each alternating series converges or diverges.

1.) $\sum_{n=1}^{\infty} \frac{n}{(-2)^{n-1}} = 1 + {}^-1 + \frac{3}{4} + \frac{^-1}{2} + \frac{5}{16} + \cdots$

$= \lim_{n\to\infty} \frac{n}{(-2)^{n-1}} = \frac{\infty}{\infty}$ $\lim_{n\to\infty} \frac{1}{(-2)^{n-1} \cdot \ln|-2|}$

$\frac{1}{\infty} = 0$

converges.

2.) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{n} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(n+1)}{n}$ diverges

$2 + {}^{-}\frac{3}{2} + \frac{4}{3} + {}^{-}\frac{5}{4} + \frac{6}{5} + \cdots$

$\lim_{n \to \infty} \frac{n+1}{n} = \frac{\infty}{\infty}$

$\lim_{n \to \infty} \frac{1}{1} = \boxed{1} \neq 0$

Approximating an Infinite Alternating Series with a Partial Sum

In most cases, we are not able to find out the actual sum of an infinite series. We can determine whether or not a series converges but we cannot determine what it converges to. For a convergent alternating series, the partial sum S_N can use a useful approximation for the sum S of a the series. The error involved is the remainder $R_N = S - S_N$. Before we can use an approximation, we need to determine "how good" that approximation is.

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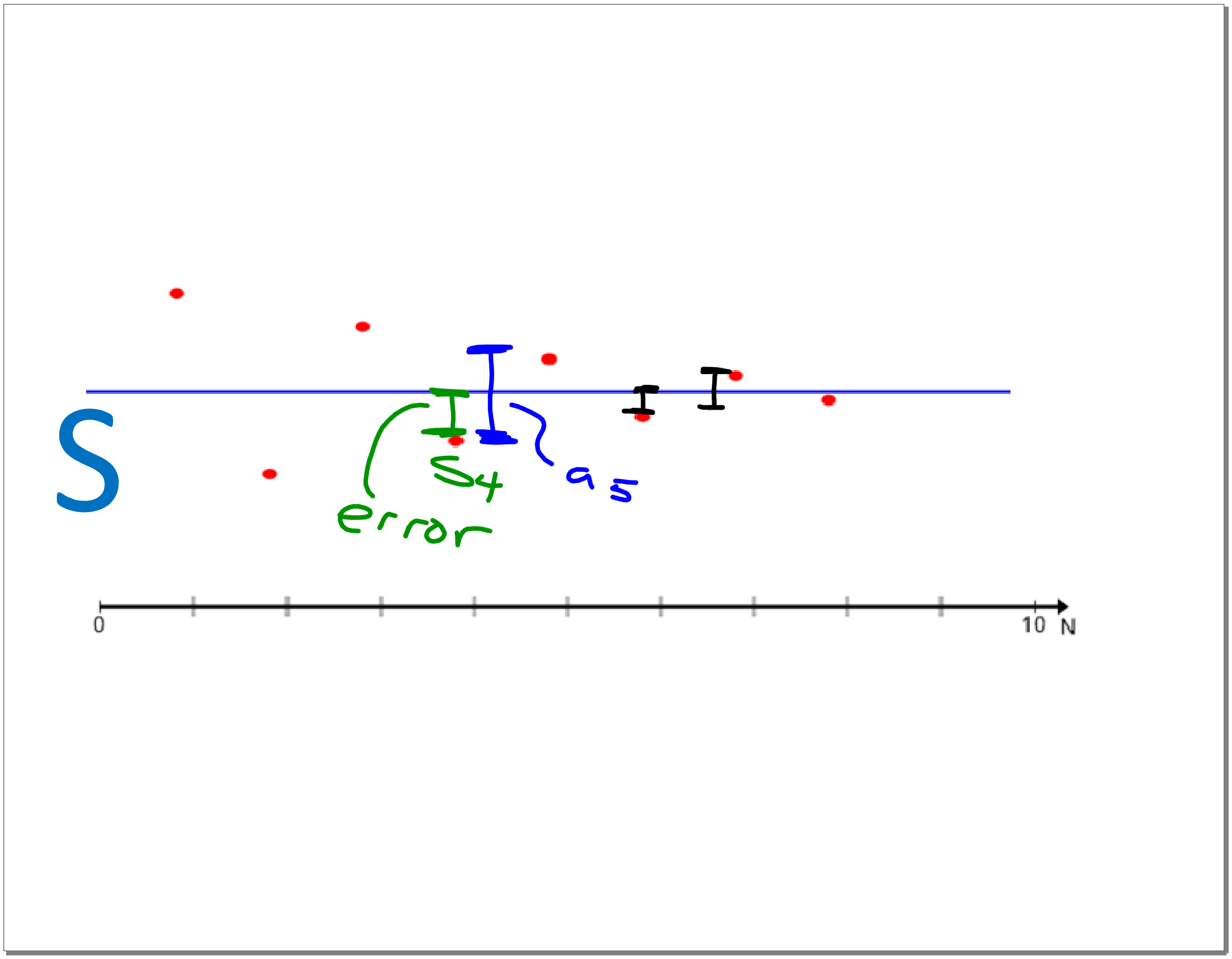

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Alternating Series Remainder Theorem

If a convergent alternating series satisfies the condition $a_{n+1}<a_n$, then the absolute value of the remainder R_N involved in approximating the sum S by S_N is no more than the 1st neglected term

$$\left|R_N\right| = \left|S - S_N\right| \leq a_{n+1}$$

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Ex2. $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{n!}\right)$

a.) Approximate the sum of the series by its first 6 terms.

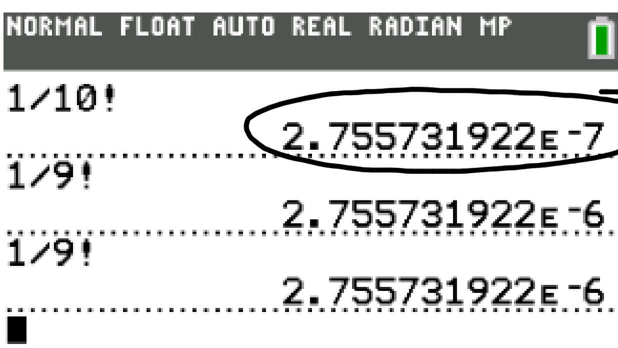
$\frac{1}{1!}+\frac{-1}{2!}+\frac{1}{3!}+\frac{-1}{4!}+\frac{1}{5!}+\frac{-1}{6!}$

$1+\frac{-1}{2}+\frac{1}{6}+\frac{-1}{24}+\frac{1}{120}+\frac{-1}{720}=\boxed{\frac{91}{144}}$

b.) How good is this approximation?

error $<\frac{1}{7!}$ error $<\frac{1}{5040}$

c.) How many terms are needed to have an error < .000001? 1×10^{-6}



error

9-terms.

Ex3. $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{n}\right)^{2}$

a.) Approximate the sum of the series by its first 8 terms.

b.) Find the error.

.012

b.) How many terms are needed to have an error < .008?

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$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2} = \frac{\sin 1}{1} + \frac{\sin 2}{4} + \frac{\sin 3}{9} + \frac{\sin 4}{16} + \frac{\sin 5}{25} \ldots$$

$$\approx .84147 + .22732 + ..01568 + -.04730 + -.03836 + \ldots$$

This series has both + and – terms but it is not really an alternating series because the signs of the terms do not alternate. How would we determine if it converges or diverges then?

Absolute and Conditional Convergence

1.) $\sum_{n=1}^{\infty} a_n$ is absolutely convergent if

$\sum_{n=1}^{\infty} |a_n|$ converges.

2.) $\sum_{n=1}^{\infty} a_n$ is conditionally convergent if

$\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges.

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$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$ converges absolutely

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$ converges conditionally

(alternating harmonic series)

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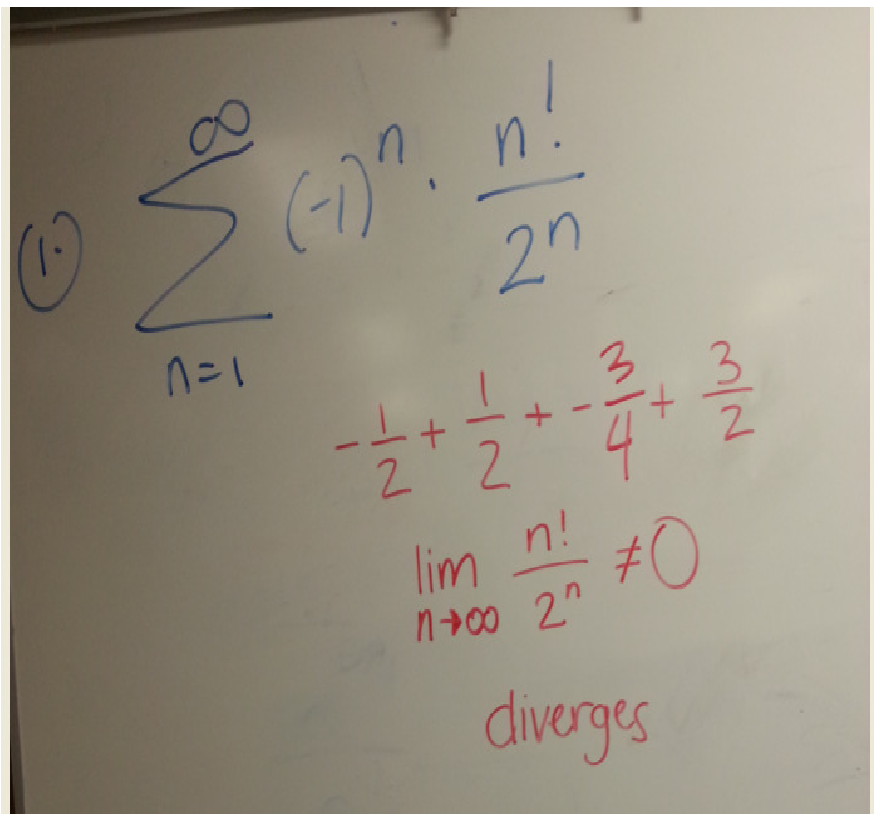
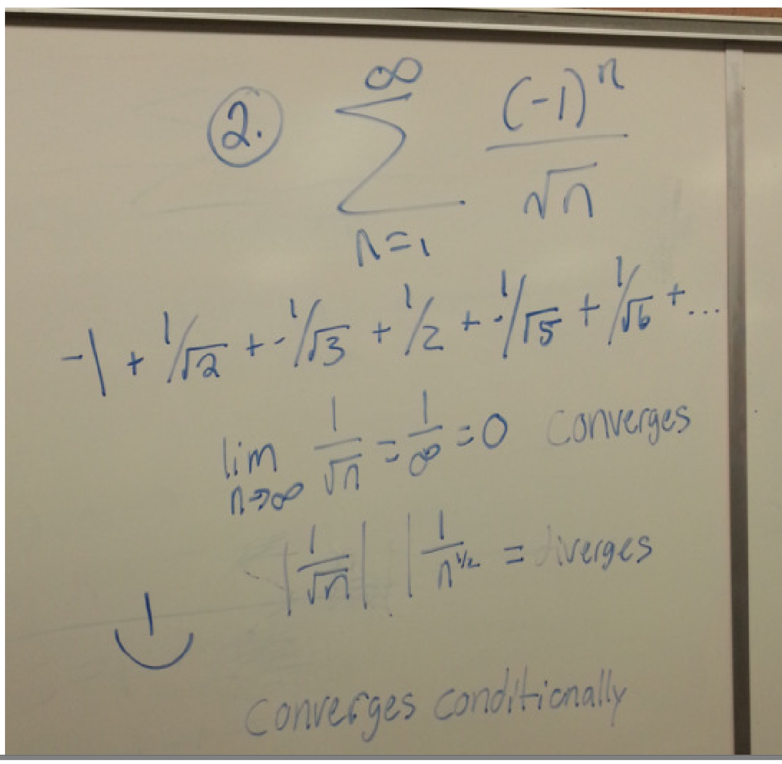
Ex4. Determine if each alternating series is convergent or divergent. Classify any convergent series as absolute or conditional.

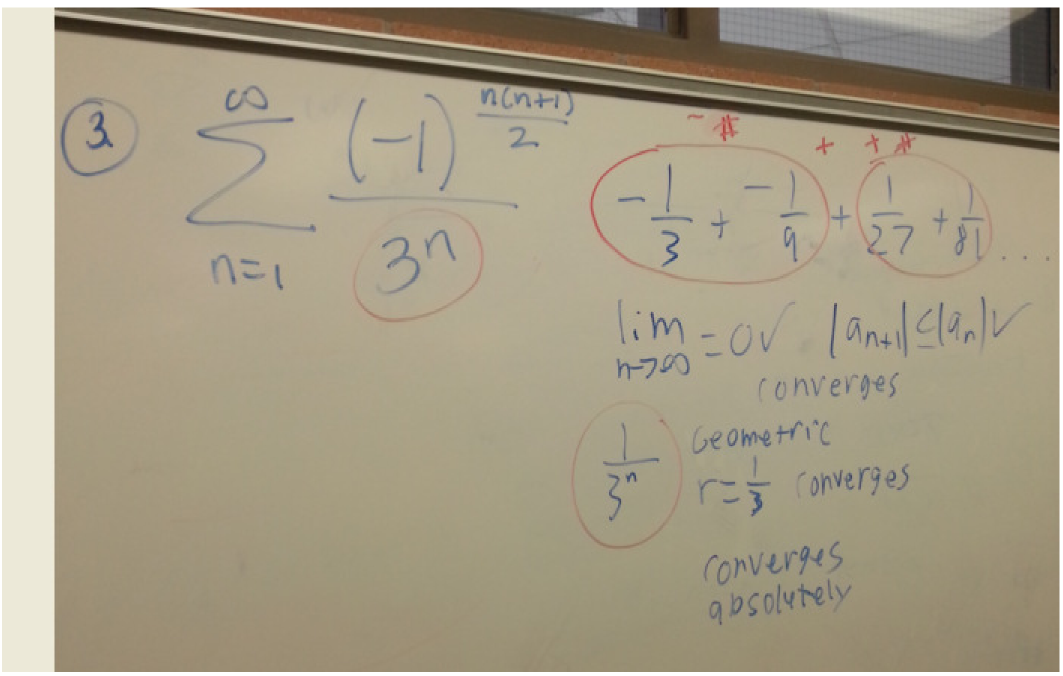
1.) $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{2^n}$



2.) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$



3.) $\sum_{n=1}^{\infty} \frac{(-1)^{\frac{n(n+1)}{2}}}{3^n}$



Homework

Alternating Series worksheet